Chapter 5

Stoichiometry and the Mole

Opening Essay

At Contrived State University in Anytown, Ohio, a new building was dedicated in March 2010 to house the College of Education. The 100,000-square-foot building has enough office space to accommodate 86 full-time faculty members and 167 full-time staff.

In a fit of monetary excess, the university administration offered to buy new furniture (desks and chairs) and computer workstations for all faculty and staff members moving into the new building. However, to save on long-term energy and materials costs, the university offered to buy only 1 laser printer per 10 employees, with the plan to network the printers together.

How many laser printers did the administration have to buy? It is rather simple to show that 26 laser printers are needed for all the employees. However, what if a chemist was calculating quantities for a chemical reaction? Interestingly enough, similar calculations can be performed for chemicals as well as laser printers.

Figure 5.1Outfitting a New Building

In filling a new office building with furniture and equipment, managers do calculations similar to those performed by scientists doing chemical reactions.
Source: Photo courtesy of Benjamin Benschneider, Cleveland State University.

We have already established that quantities are important in science, especially in chemistry. It is important to make accurate measurements of a variety of quantities when performing experiments. However, it is also important to be able to relate one measured quantity to another, unmeasured quantity. In this chapter, we will consider how we manipulate quantities to relate them to each other.

5.1 Stoichiometry

LEARNING OBJECTIVES

1. Define *stoichiometry*.
2. Relate quantities in a balanced chemical reaction on a molecular basis.

Consider a classic recipe for pound cake: 1 pound of eggs, 1 pound of butter, 1 pound of flour, and 1 pound of sugar. (That's why it's called "pound cake.") If you have 4 pounds of butter, how many pounds of sugar, flour, and eggs do you need? You would need 4 pounds each of sugar, flour, and eggs.

Now suppose you have 1.00 g H_2. If the chemical reaction follows the balanced chemical equation

$$2H_2(g) + O_2(g) \rightarrow 2H_2O(\ell)$$

then what mass of oxygen do you need to make water?

Curiously, this chemical reaction question is very similar to the pound cake question. Both of them involve relating a quantity of one substance to a quantity of another substance or substances. The relating of one chemical substance to another using a balanced chemical reaction is called **stoichiometry**. Using stoichiometry is a fundamental skill in chemistry; it greatly broadens your ability to predict what will occur and, more importantly, how much is produced.

Let us consider a more complicated example. A recipe for pancakes calls for 2 cups (c) of pancake mix, 1 egg, and 1/2 c of milk. We can write this in the form of a chemical equation:

$$2 \text{ c mix} + 1 \text{ egg} + 1/2 \text{ c milk} \rightarrow 1 \text{ batch of pancakes}$$

If you have 9 c of pancake mix, how many eggs and how much milk do you need? It might take a little bit of work, but eventually you will find you need 4½ eggs and 2¼ c milk.

How can we formalize this? We can make a conversion factor using our original recipe and use that conversion factor to convert from a quantity of one substance to a quantity of another substance, similar to the way we constructed a conversion factor between feet and yards in http://catalog.flatworldknowledge.com/bookhub/reader/2273 - ball-ch02. Because one recipe's worth of pancakes requires 2 c of pancake mix, 1 egg, and 1/2 c of milk, we actually have the following mathematical relationships that relate these quantities:

2 c pancake mix ⇔ 1 egg ⇔ 1/2 c milk

where ⇔ is the mathematical symbol for "is equivalent to." This does not mean that 2 c of pancake mix equal 1 egg. However, *as far as this recipe is concerned*, these are the equivalent quantities needed for a single recipe of pancakes. So, any possible quantities of two or more ingredients must have the same numerical ratio as the ratios in the equivalence.

We can deal with these equivalences in the same way we deal with equalities in unit conversions: we can make conversion factors that essentially equal 1. For example, to determine how many eggs we need for 9 c of pancake mix, we construct the conversion factor

$$\frac{1\ \text{egg}}{2\ \text{c pancake mix}}$$

This conversion factor is, in a strange way, equivalent to 1 because the recipe relates the two quantities. Starting with our initial quantity and multiplying by our conversion factor,

$$9\ \cancel{\text{c pancake mix}} \times \frac{1\ \text{egg}}{2\ \cancel{\text{c pancake mix}}} = 4.5\ \text{eggs}$$

Note how the units *cups pancake mix* canceled, leaving us with units of *eggs*. This is the formal, mathematical way of getting our amounts to mix with 9 c of pancake mix. We can use a similar conversion factor for the amount of milk:

$$9\ \cancel{\text{c pancake mix}} \times \frac{1/2\ \text{c milk}}{2\ \cancel{\text{c pancake mix}}} = 2.25\ \text{c milk}$$

Again, units cancel, and new units are introduced.

A balanced chemical equation is nothing more than *a recipe for a chemical reaction*. The difference is that a balanced chemical equation is written in terms of atoms and molecules, not cups, pounds, and eggs.

For example, consider the following chemical equation:

$$2H_2(g) + O_2(g) \rightarrow 2H_2O(\ell)$$

We can interpret this as, literally, "two hydrogen molecules react with one oxygen molecule to make two water molecules." That interpretation leads us directly to some equivalences, just as our pancake recipe did:

$$2H_2 \text{ molecules} \Leftrightarrow 1O_2 \text{ molecule} \Leftrightarrow 2H_2O \text{ molecules}$$

These equivalences allow us to construct conversion factors:

$$\frac{2 \text{ molecules } H_2}{1 \text{ molecule } O_2} \quad \frac{2 \text{ molecules } H_2}{2 \text{ molecules } H_2O} \quad \frac{1 \text{ molecule } O_2}{2 \text{ molecules } H_2O}$$

and so forth. These conversions can be used to relate quantities of one substance to quantities of another. For example, suppose we need to know how many molecules of oxygen are needed to react with 16 molecules of H_2. As we did with converting units, we start with our given quantity and use the appropriate conversion factor:

$$16 \cancel{\text{ molecules } H_2} \times \frac{1 \text{ molecule } O_2}{2 \cancel{\text{ molecules } H_2}} = 8 \text{ molecules } O_2$$

Note how the unit *molecules* H_2 cancels algebraically, just as any unit does in a conversion like this. The conversion factor came directly from the coefficients in the balanced chemical equation. This is another reason why a properly balanced chemical equation is important.

EXAMPLE 1

How many molecules of SO_3 are needed to react with 144 molecules of Fe_2O_3 given this balanced chemical equation?

$Fe_2O_3(s) + 3SO_3(g) \rightarrow Fe_2(SO_4)_3$

Solution

We use the balanced chemical equation to construct a conversion factor between Fe_2O_3 and SO_3. The number of molecules of Fe_2O_3 goes on the bottom of our conversion factor so it cancels with our given amount, and the molecules of SO_3 go on the top. Thus, the appropriate conversion factor is

$$\frac{3 \text{ molecules } SO_3}{1 \text{ molecule } Fe_2O_3}$$

Starting with our given amount and applying the conversion factor, the result is

$$144 \cancel{\text{ molecules } Fe_2O_3} \times \frac{3 \text{ molecules } SO_3}{1 \cancel{\text{ molecule } Fe_2O_3}} = 432 \text{ molecules } SO_3$$

We need 432 molecules of SO_3 to react with 144 molecules of Fe_2O_3.

Test Yourself

How many molecules of H_2 are needed to react with 29 molecules of N_2 to make ammonia if the balanced chemical equation is $N_2 + 3H_2 \rightarrow 2NH_3$?

Answer

87 molecules

Chemical equations also allow us to make conversions regarding the number of atoms in a chemical reaction because a chemical formula lists the number of atoms of each element in a compound. The formula H_2O indicates that there are two hydrogen atoms and one oxygen atom in each molecule, and these relationships can be used to make conversion factors:

$$\frac{2\ \text{atoms H}}{1\ \text{molecule H}_2\text{O}} \qquad \frac{1\ \text{molecule H}_2\text{O}}{1\ \text{atom O}}$$

Conversion factors like this can also be used in stoichiometry calculations.

EXAMPLE 2

How many molecules of NH_3 can you make if you have 228 atoms of H_2?

Solution

From the formula, we know that one molecule of NH_3 has three H atoms. Use that fact as a conversion factor:

$$228\ \cancel{\text{atoms H}} \times \frac{1\ \text{molecule NH}_3}{3\ \cancel{\text{atoms H}}} = 76\ \text{molecules NH}_3$$

Test Yourself

How many molecules of $Fe_2(SO_4)_3$ can you make from 777 atoms of S?

Answer

259 molecules

KEY TAKEAWAY

- Quantities of substances can be related to each other using balanced chemical equations.

EXERCISES

1. Think back to the pound cake recipe. What possible conversion factors can you construct relating the components of the recipe?

2. Think back to the pancake recipe. What possible conversion factors can you construct relating the components of the recipe?

3. What are all the conversion factors that can be constructed from the balanced chemical reaction $2H_2(g) + O_2(g) \rightarrow 2H_2O(\ell)$?

4. What are all the conversion factors that can be constructed from the balanced chemical reaction $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$?

5. Given the chemical equation
   $Na(s) + H_2O(\ell) \rightarrow NaOH(aq) + H_2(g)$
   a. Balance the equation.
   b. How many molecules of H_2 are produced when 332 atoms of Na react?

6. Given the chemical equation
   $S(s) + O_2(g) \rightarrow SO_3(g)$
   a. Balance the equation.
   b. How many molecules of O_2 are needed when 38 atoms of S react?

7. For the balanced chemical equation
   $6H^+(aq) + 2MnO_4^-(aq) + 5H_2O_2(\ell) \rightarrow 2Mn^{2+}(aq) + 5O_2(g) + 8H_2O(\ell)$

how many molecules of H_2O are produced when 75 molecules of H_2O_2 react?

8. For the balanced chemical reaction

   $2C_6H_6(\ell) + 15O_2(g) \rightarrow 12CO_2(g) + 6H_2O(\ell)$

   how many molecules of CO_2 are produced when 56 molecules of C_6H_6 react?

9. Given the balanced chemical equation

   $Fe_2O_3(s) + 3SO_3(g) \rightarrow Fe_2(SO_4)_3$

   how many molecules of $Fe_2(SO_4)_3$ are produced if 321 atoms of S are reacted?

10. For the balanced chemical equation

    $CuO(s) + H_2S(g) \rightarrow CuS + H_2O(\ell)$

    how many molecules of CuS are formed if 9,044 atoms of H react?

11. For the balanced chemical equation

    $Fe_2O_3(s) + 3SO_3(g) \rightarrow Fe_2(SO_4)_3$

    suppose we need to make 145,000 molecules of $Fe_2(SO_4)_3$. How many molecules of SO_3do we need?

12. One way to make sulfur hexafluoride is to react thioformaldehyde, CH_2S, with elemental fluorine:

    $CH_2S + 6F_2 \rightarrow CF_4 + 2HF + SF_6$

    If 45,750 molecules of SF_6 are needed, how many molecules of F_2 are required?
13. Construct the three independent conversion factors possible for these two reactions:

a. $2H_2 + O_2 \rightarrow 2H_2O$

b. $H_2 + O_2 \rightarrow H_2O_2$

Why are the ratios between H_2 and O_2 different?

The conversion factors are different because the stoichiometries of the balanced chemical reactions are different.

13. Construct the three independent conversion factors possible for these two reactions:

a. $2Na + Cl_2 \rightarrow 2NaCl$

b. $4Na + 2Cl_2 \rightarrow 4NaCl$

What similarities, if any, exist in the conversion factors from these two reactions?

ANSWERS

1. $\frac{2\ \text{molecules H}_2}{1\ \text{molecule O}_2}, \frac{1\ \text{molecule O}_2}{2\ \text{molecules H}_2\text{O}}, \frac{2\ \text{molecules H}_2}{2\ \text{molecules H}_2\text{O}}$ are two conversion factors that can be constructed from the pound cake recipe. Other conversion factors are also possible.

3. $\frac{2\ \text{molecules H}_2}{1\ \text{molecule O}_2}, \frac{1\ \text{molecule O}_2}{2\ \text{molecules H}_2\text{O}}, \frac{2\ \text{molecules H}_2}{2\ \text{molecules H}_2\text{O}}$, and their reciprocals are the conversion factors that can be constructed.

5. a. $2Na(s) + 2H_2O(\ell) \rightarrow 2NaOH(aq) + H_2(g)$

   b. 166 molecules

7. 120 molecules

9. 107 molecules

11. 435,000 molecules

13.

a. $\frac{2 \text{ molecules } H_2}{1 \text{ molecule } O_2}$, $\frac{1 \text{ molecule } O_2}{2 \text{ molecules } H_2O}$, and $\frac{2 \text{ molecules } H_2}{2 \text{ molecules } H_2O}$

b. $\frac{1 \text{ molecule } H_2}{1 \text{ molecule } O_2}$, $\frac{1 \text{ molecule } O_2}{1 \text{ molecule } H_2O_2}$, and $\frac{1 \text{ molecule } H_2}{1 \text{ molecule } H_2O_2}$